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Gravitationally Induced Electric Field in Metals†

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The gravitationally induced electric field in a metal is calculated through the electron-phonon interaction. The field is of magnitude $\sim 10^{-6}$ V/m and is directed upward, in agreement with the results of Dessler *et al.* The electric field also exists inside superconducting materials, since once the field is decoupled, the electron-phonon interaction is essentially unchanged. This implies, of course, that other electron-phonon effects (e.g., resistivity) are unaffected by either the gravitational field or the induced electric field.

I. INTRODUCTION

Recently, there has been interest in an electric field in metals induced by a gravitational field. This interest divides between the related problems of a gravitationally induced electric field inside a metal and an induced field outside the surface of a metal. The situation is complicated, however, by the fact that the internal electric field is easier to calculate than the external field, which depends on the behavior of the surface dipole moment. On the other hand, measurement of the external field, although difficult, is easier than measurement of the internal field which is not readily accessible. This work is a calculation of the internal gravitationally induced electric field.

Attempts have been made to calculate the field outside a metal[1-3] and a inside metal.[2,4] Measurements of the field outside a metal have been attempted.[5-7] Schiff and Barnhill[1] obtained $mg/e \simeq 10^{-10}$ V/m directed downward for the field outside the metal, which is just the field required to screen out the gravitational force. In their analysis, however, they neglected the gravitational compression of the lattice. Dessler, Michel, Rorschach, and Trammel[2] recalculated the electric field including the elastic compression of the solid under its own weight, and related the change in density of the electron gas to the resulting density gradient of the ions. By requiring that the electrochemical potential of the electrons be constant, they derived the internal electric field necessary to balance the pressure gradient due to the inhomogeneous electron density. This is of strength $\sim 10^{-6}$ V/m and oppositely directed to the Schiff-Barnhill result. For the field outside the metal they obtained a similar result. Herring[3] considered the gravitationally induced electric field outside a body by carefully treating the surface stresses. He concluded that the larger Dessler *et al.* (DMRT) field exists outside a metal.

Still another, even simpler method, using the Fermi-Thomas model, has been suggested by Peshkin,[8] and leads to the DMRT field inside a metal.

In this analysis, a different method is used to

calculate the gravitationally induced electric field. The Hamiltonian for the solid is written down in full, and the effect of the gravitational compression of the lattice is taken into account through the electron-phonon interaction. The Hamiltonian can be divided into two parts; one produces the electric field of interest here, and the other is an interaction of the electrons with the phonons of the gravitationally compressed lattice. This treatment is directly applicable to a superconductor, and after accounting for the gravitationally induced electric field, an electron-phonon interaction remains which is virtually unchanged from that considered in the usual treatment of superconductivity without gravitation. This interaction then produces the superconducting state exactly as if there were no gravitational field or induced electric field. The method thus separates the effects of a gravitational field from other electron-phonon effects in an explicit fashion. In particular, the analysis implies that the internal electric field also exists inside superconducting materials. Here too, of course, there is no net force on the electrons.

The results of this calculation agree with those of DMRT[2] for the internal field.

II. LATTICE HAMILTONIAN

The normal-mode functions for the solid are assumed to be plane waves in the horizontal directions (x-y) and a sine wave in the vertical (z) direction, corresponding to the vibrations of a body fixed in place at the bottom and free at all other surfaces. These asymmetric boundary conditions are needed if the body is not to fall freely in a gravitational field. Instead of simple plane waves, the vibrational modes are to be a product of the plane wave and a sine wave. The normal mode with wave vector $\vec{q}$ is $F_{\vec{q}}(\vec{x})$,

$$F_{\vec{q}}(\vec{x}) = e^{i(q_x x + q_y y)} \sin(q_z z) \ . \tag{1}$$

The origin of the coordinate system is at the bottom of the body and the wave vectors are given by the usual integral multiples of $2\pi/L$ for x and y and odd multiples of $\pi/2L$ for z. The odd multiples of $\pi/2L$ in the vertical direction are necessary to satisfy boundary conditions of a free upper surface and a fixed lower surface,

$$\begin{aligned} q_{x,y} &= 2\pi m_{x,y}/L_{x,y}, \quad m_{x,y} = 0, \pm 1, \pm 2, \ldots \\ q_z &= (2l+1)\pi/2L_z, \qquad l = 0, 1, 2, \ldots \ . \end{aligned} \tag{2}$$

The lattice Hamiltonian is written in terms of phonons:

$$H_{\rm ph} = \sum_{q\lambda} \hbar\omega_{q\lambda}(a^\dagger_{q\lambda} a_{q\lambda} + \tfrac{1}{2}) \ . \tag{3}$$

$a_{q\lambda}$ destroys a phonon of wave vector $\vec{q}$ and polarization λ. The operators a, $a^\dagger$ obey the usual boson commutation rules.

In terms of $a_{q\lambda}$ and $a^\dagger_{q\lambda}$ the displacement operator for the ions is given by the usual expression

$$\vec{u}(\vec{x}) = \sum_{q\lambda} \hat{\epsilon}_{q\lambda} \left(\frac{\hbar}{mN\omega_{q\lambda}}\right)^{1/2} (a_{q\lambda} + a^\dagger_{-q\lambda}) F_{\vec{q}}(\vec{x}) \ . \tag{4}$$

$\hat{\epsilon}_{q\lambda}$ is a polarization unit vector, M is the ionic mass, and N is the number of ions.

If the ith ion is at height z_i, the gravitational Hamiltonian is the sum of the gravitational energy of each ion Mgz_i:

$$H_{\rm grav} = \sum_i Mgz_i = \sum_i Mg[z_i^0 + \vec{u}(z_i)\cdot\hat{z}] \ . \tag{5}$$

The sum over equilibrium sites is a constant equal to $\frac{1}{2}NML_z$. The sum over lattice sites using expression (4) for $\vec{u}$ yields an expression for $H_{\rm grav}$ in second-quantized form. The absence of any x or y dependence in (5) implies that $q_x = q_x = 0$ only:

$$H_{\rm grav} = g\frac{(MN\hbar)^{1/2}}{L_z} \sum_{q_z} \omega_{q_l}^{-1/2} \frac{1}{q_z} (a_{q_z l} + a^\dagger_{q_z l}) \ . \tag{6}$$

The subscript l refers to longitudinal phonons.

Equations (3) and (6) can be combined, and a transformation made to new boson operators:

$$H_{\rm ph} + H_{\rm grav} = \sum_{q\lambda} \hbar\omega_{q\lambda}(A^\dagger_{q\lambda} A_{q\lambda} + \tfrac{1}{2}) + E_g + \tfrac{1}{2}NML_z \ , \tag{7}$$

$$A_{q_\lambda} = a_{q_\lambda} + \delta_{q_x 0}\delta_{q_y 0}\delta_{\lambda l} \frac{g}{L_z}\left(\frac{MN}{\hbar}\right)^{1/2} \omega_{q_z l}^{-3/2} \frac{1}{q_z}, \tag{8}$$

$$E_g = -16MNL_z^2 g^2/c_l^2\pi^4 \ , \tag{9}$$

where c_l is the longitudinal sound velocity. These new boson operators [Eq. (8)] correspond to allowing the ions to come to their new gravitationally compressed equilibrium positions.

The Hamiltonian of the electrons is also written in second-quantized form

$$\begin{aligned} H_e &= \sum_{ks} \epsilon_k c^\dagger_{k_s} c_{k_s} \ , \\ \epsilon_k &= \frac{\hbar^2 k^2}{2m}, \quad k_i = \frac{2\pi M_i}{L_i}, \quad M_i = 0, \pm 1, \ldots \ . \end{aligned} \tag{10}$$

The c's satisfy the fermion anticommutation rules.

III. ELECTRON-PHONON INTERACTION

The interaction of the electrons and phonons is assumed to arise from the polarization of the electron gas caused by the ionic vibrations. The coupling between the electron density and polarization is assumed to be screened and is of short range.[9]

$$H_{\rm el\text{-}ph} = \int d^3r\, d^3r'\, (-e)\rho(r)\vec{\nabla}'\cdot\vec{\mathcal{P}}(\vec{r}\,')K(r-r') \ , \tag{11}$$

$\rho(r) = (1/v)\sum_k \rho_k e^{i\vec{k}\cdot\vec{r}}$ is the electron density, $\rho_k = \sum_{k'} c^\dagger_{k'-k} c_{k'}$, $\vec{\mathcal{P}}(\vec{r}\,') \equiv \xi(-e)NZ\,\vec{U}(\vec{r}\,')/v$ is the electron polarization, ξ is an empirical constant, Z

is the ionic valence, $K(r-r')=(1/k_c^2)\,\delta(\vec{r}-\vec{r}\,')$ is the short-range coupling, and k_c is the screening wave vector. Using (4) for $\vec{u}(\vec{r}\,')$ and performing the double integrals in (11) gives $H_{\text{el-ph}}$ in the desired form

$$H_{\text{el-ph}}=\frac{iNZe^2\zeta}{2vL_z}\frac{1}{k_c^2}\sum_{qk}\delta_{k_x,-q_x}\delta_{k_y,-q_y}\left(\frac{\hbar}{MN\omega_{q_\lambda}}\right)^{1/2}\rho_k \times(a_{q_\lambda}+a^{\dagger}_{-q_\lambda})\hat{\epsilon}_{q_\lambda}\cdot[\alpha\vec{q}_\perp+\hat{z}\beta q_z]\ , \tag{12}$$

$$q_\perp=\hat{x}q_x+\hat{y}q_y\ ,$$

$$\alpha=\frac{1-e^{i(k_z+q_z)L_z}}{k_z+q_z}-\frac{1-e^{i(k_z-q_z)L_z}}{k_z-q_z}\ , \tag{13}$$

$$\beta=\frac{1-e^{i(k_z+q_z)L_z}}{k_z+q_z}+\frac{1-e^{i(k_z-q_z)L_z}}{k_z-q_z}\ , \tag{14}$$

$k_z=2\pi M'/L_z$ $M=0,\ \pm1,\ \pm2,\ldots$; an electron wave vector $q_z=(2l+1)\pi/2L_z$, $l=0,1,2,\ldots$; an ionic wave vector.

The empirical constant ζ defined in (11) is evaluated by calculating the lattice resistivity of the material with this form for the electron-phonon interaction. ζ/k_c^2 enters in both the resistivity and the internal electric field, so the value of the field is independent of the screening length chosen. Using an inverse Wigner-Seitz radius implies $\zeta\cong 2\pi$ for most metals.

IV. GRAVITATIONALLY INDUCED ELECTRIC FIELD

The canonical transformation defined by (8) is now made in expression (12). The result has two parts. The first is an expression just like (12) except that the a's are replaced by A's. This is the interaction between the electrons and the phonons of the gravitationally compressed lattice. This term will lead to superconductivity (and all other effects which depend on the electron-phonon interaction) in the usual way. The second part does not contain any phonon operators; it comes from the nonoperator terms in (8). This term H_2 is a one-body potential on the electrons,

$$H_2=G\sum_{k_z}\rho_{k_z}\left(\sum_{q_z}\frac{1-e^{i(k_z+q_z)L_z}}{q_z^2(k_z+q_z)}+\frac{1-e^{i(k_z-q_z)L_z}}{q_z^2(k_z-q_z)}\right), \tag{15}$$

$$G=-i\frac{NZe^2g\zeta}{VL_z^2c_l^2k_c^2}\ .$$

The term in the square brackets in (15), a sum over phonon wave vectors, can be done approximately, and is called D:

$$D=\left(\frac{2L_z}{\pi}\right)^3\sum_{l=0}^{N_z}\left[\frac{1}{(2l+1)^2(2l+1+4M)}-\frac{1}{(2l+1)^2(2l+1-4M)}-i\left(\frac{(-1)^l}{(2l+1)^2(2l+1+4M)}+\frac{(-1)^l}{(2l+1)^2(2l+1-4M)}\right)\right], \tag{16}$$

where N_z is the number of crystal planes in the z direction, i.e., $N_z\cong(N)^{1/3}$, $M=0,\ \pm1,\ \pm2\ldots$, electron quantum numbers. This can be written

$$D=\left(\frac{2L_z}{\pi}\right)^3\sum_l\left\{i(-1)^l\left[\frac{1}{8M^2}\frac{1}{2l+1}-\frac{1}{16M^2}\left(\frac{1}{2l+1+4M}+\frac{1}{2l+1-4M}\right)\right]+\left[\frac{1}{2M}\frac{1}{(2l+1)^2}+\frac{1}{16M^2}\times\left(\frac{1}{2l+1+4M}-\frac{1}{2l+1-4M}\right)\right]\right\}\ . \tag{17}$$

The most important term in (17) is the one which depends on $1/M$. Keeping only this term and ignoring the rest is equivalent to saying that those electrons which are important are those with large k, i.e., large energy and short range. The long-range phonons, on the other hand, must be treated carefully since they describe the gravitational compression of the lattice,

$$D\cong\left(\frac{2L_z}{\pi}\right)^3\frac{1}{2M}\sum_l\frac{1}{(2l+1)^2}\cong\left(\frac{2L_z}{\pi}\right)^3\frac{1}{2M}\frac{\pi^2}{8}\ . \tag{18}$$

Substitution in (15) yields an expression for H_2,

$$H_2\cong-L_z^2G\sum_{k_c\neq0}\frac{1}{k_z}\rho_{-k_z}\ . \tag{19}$$

In (19) the coefficient of ρ_{-k_z} is the Fourier transform of the one-body potential acting on the electrons. It is proportional to the Fourier transform of z,

$$F(z)=i\frac{1}{k_z}\delta_{k_x0}\delta_{k_y0},\quad k_z\neq0\ . \tag{20}$$

The one-body potential in configuration space $\varphi(z)$ is therefore linear in z,

$$\varphi(z)\cong(iGL_z^2)z\ , \tag{21}$$

$$\varphi(z) \cong (NZe^2 g\zeta / V c_l^2 k_c^2)\, z \ . \tag{22}$$

For c_l, the longitudinal sound velocity, the expression corresponding to a continuum with Young's modulus Y, Poisson's ratio σ, and mass density MN/v were used:

$$c_l^2 = \frac{Y(1-\sigma)}{(MN/v)(1+\sigma)(1-2\sigma)} \ . \tag{23}$$

The internal electric field is given by the gradient of $\varphi(z)$,

$$\vec{E}_{\rm int} = -\vec{\nabla}\left(\frac{1}{-e}\,\varphi(z)\right) = \frac{1}{e}\vec{\nabla}\varphi \ ,$$
$$\vec{E}_{\rm int} = \hat{z}\,\frac{N^2}{V^2}\,\frac{ZeMg}{k_c^2}\,\frac{(1+\sigma)(1-2\sigma)}{Y(1-\sigma)} \ . \tag{24}$$

Table I has some typical values of $\vec{E}_{\rm int}$.

Since the long-range phonons are mainly responsible for describing the gravitational compression of the lattice, perhaps a better value for ζ, the electron-phonon coupling constant, can be obtained from ultrasonic attenuation data than from resistivity data. The magnitude of the gravity-induced electric field is roughly an order of magnitude smaller than the values given in Table I, if ultrasonic data are used.

TABLE I. Typical values of $E_{\rm int}$.

Material	$E_{\rm int}$(V/m)
Cu	4.0×10^{-6}
Sn	6.2×10^{-6}
Pb	1.1×10^{-5}
Al	4.0×10^{-6}

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Equilibrium Defect Concentration in Crystalline Lithium

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The linear thermal expansion of pure lithium has been measured from 0 °C to the melting point by dilatometric and x-ray methods. Changes in length and lattice parameter were determined, respectively, by means of a Fizeau-type precision interferometer and by a high-angle backreflection x-ray technique. The two expansion curves diverge above 65 °C in a manner indicating the predominance of vacancy-type defects. If the divergence of the two curves is assumed to be due only to monovacancies, the results yield a formation energy $E_{1v}^f = 0.34 \pm 0.04$ eV and a formation entropy $S_{1v}^f = (0.9 \pm 0.8)k$. The value of E_{1v}^f accounts for 0.63 of the activation energy for self-diffusion. This result is much smaller than the corresponding ratio for Na, and, in fact, is comparable to results reported for fcc metals. The results are discussed in terms of the possible coexistence of monovacancies with either divacancies or interstitials.

I. INTRODUCTION

The x-ray dilatometric method employed in this work has previously been used to determine the equilibrium vacancy concentration in the following fcc metals: Ag,[1] Al,[2] Au,[3] Cu,[4] and Pb.[5] Although the vacancy concentration at the melting point varies between 1.7×10^{-4} and 9.4×10^{-4} (mole fraction),